Math Superbowl XLVII
Level 6 Blitz: Calculus – 2022

Directions: Select the most correct answer for each question and mark it on your answer form. No calculators of any sort are allowed.

1. $\lim_{x\to 2}\left(2x^3+x-3\right)=$

   a) 15 b) 14 c) 13
   d) 12 e) None of these

2. $\lim_{x\to 3}\dfrac{x^3-27}{x^2-9}=$

   a) Does not exist b) 3 c) 4
   d) ∞ e) None of these

3. $\lim_{x\to 0} x^2\sin\left(\dfrac{1}{x^2}\right)=$

   a) Does not exist b) ∞ c) 1
   d) 0 e) None of these

4. $\lim_{t\to 2}\dfrac{\sqrt{t^2+5}-3}{t-2}=$

   a) Does not exist b) ∞ c) $\dfrac{2}{3}$
   d) $\dfrac{1}{6}$ e) None of these

5. $\lim_{x\to 0}\dfrac{1}{x}\sin(x)=$

   a) Does not exist b) ∞ c) 1
   d) 0 e) None of these

6. $\lim_{h\to 0} \dfrac{\cos(h)-1}{h} =$

a) 1 b) 0 c) $\dfrac{\sqrt{2}}{2}$

d) –1 e) None of these

7. An open-topped box is to be constructed from a square piece of cardboard with edge length 12 in. Four squares each having edge length x inches are cut from the four corners of the cardboard. Then fold up the remaining flaps. The volume V of the box (in cubic inches) as a function of x is $V(x) =$

a) $x(12-x)^2$ b) $4x(6-x)^2$ c) $2x(12-2x)^2$

d) $4x(25-x^2)$ e) None of these

8. $\lim_{v\to 1^-} \dfrac{v-1}{|v-1|} =$

a) 1 b) –1 c) Does not exist

d) ∞ e) None of these

9. $\lim_{x\to 2} \dfrac{1}{x-2} =$

a) ∞ b) 0 c) $-\infty$

d) Does not exist e) None of these

10. Let $f(x) = \dfrac{1}{\sqrt[3]{x^2}}$. Then $f'(1) =$

a) 1 b) –1 c) $\dfrac{2}{3}$

d) $-\dfrac{2}{3}$ e) None of these

11. A ball is thrown vertically up from the ground with a velocity of 64 ft/s. Its height h (in feet) above the ground after t seconds is $h = 64t - 16t^2$. What is the velocity of the ball when it is traveling upwards at a height of 48 feet?

a) 8 ft/s b) 16 ft/s c) 24 ft/s

d) 32 ft/s e) None of these

12. Find the derivative of $f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$.

a) $\dfrac{1 + \sqrt{x}}{2\sqrt{x + \sqrt{x}}}$

b) $\dfrac{2\sqrt{x} + 1}{4\sqrt{x^2 + x\sqrt{x}}}$

c) $\dfrac{1}{2}(1 + \sqrt{x})$

d) $\dfrac{4\sqrt{x^2 + x\sqrt{x}} + 2\sqrt{x} + 1}{8\sqrt{x + \sqrt{x + \sqrt{x}}}\sqrt{x + \sqrt{x}}\sqrt{x}}$

e) None of these

13. Find a formula for $f^{(n)}(x)$ if $f(x) = \dfrac{1}{x}$.

a) $f^{(n)}(x) = (-1)^n \dfrac{n!}{x^{n+1}}$

b) $f^{(n)}(x) = (-1)^{n+1} \dfrac{n!}{x^{n+1}}$

c) $f^{(n)}(x) = (-1)^{n-1} \dfrac{(n+1)!}{x^n}$

d) $f^{(n)}(x) = (-1)^n \dfrac{(n-1)!}{x^n}$

e) None of these

14. If $y = \ln(xy)$, find y'.

a) Cannot be found

b) $y' = \dfrac{x + y}{xy}$

c) $y' = \dfrac{\ln(xy)}{x(\ln(xy) - 1)}$

d) $y' = \dfrac{1}{xy}$

e) None of these

15. If $y = (\cos x)^x$, where $0 < x < \pi/2$, find y'.

a) $y' = x(\sin x)^{x-1}$

b) $y' = (\cos x)^x(\ln \cos x - x \tan x)$

c) $y' = x^2(\cos x)^{x-1}$

d) $y' = (\sin x)^x(x \cot x + \ln \sin x)$

e) None of these

16. Water fills a cylindrical container at a rate of 20 in^3/min. The circular base of the container has a radius of 4 in. How fast is the height of the water increasing?

a) $\sqrt{20}$ ft/min

b) $\dfrac{5}{4\pi}$ ft/min

c) 10 ft/min

d) 2π ft/min

e) None of these

17. A lighthouse is located on a small island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

a) $\frac{80\pi}{3}$ km/min b) $\frac{80}{9}$ km/min c) $\frac{40}{9}$ km/min

d) $\frac{40\pi}{3}$ km/min e) None of these

18. If $y = x^3 + x^2 - x + 1$, then the relative maximum value occurs at

a) $\frac{1}{3}$ b) 1 c) $-\frac{1}{3}$

d) -1 e) None of these

19. The absolute maximum value for $f(x) = \frac{x}{2} + \sin x$, where $0 \leq x \leq \pi$, is

a) $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ b) $\frac{\pi}{2}$ c) $\frac{2\pi}{3} + \frac{1}{2}$

d) $\frac{2\pi}{3}$ e) None of these

20. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1 + \frac{3i}{n}} =$

a) Cannot be found b) $\frac{2}{3}$ c) $\frac{14}{3}$

d) $\frac{4}{3}$ e) None of these

21. If c is a constant and the equation $2x^5 + 3x^3 + c = 0$ has at most n real roots, then $n =$

a) 1 b) 2 c) 3

d) 4 e) None of these

22. $\lim_{x\to\infty} \frac{e^x}{x^3} =$

a) $\frac{1}{6}$ b) 0 c) ∞

d) $\frac{1}{2}$ e) None of these

23. $\lim_{x\to\infty} \left(e^{-2x}\cos(x)\right) =$

a) e^{-2} b) ∞ c) 0

d) Does not exist e) None of these

24. The graph of $f(x) = \frac{x^2+1}{x}$ has

a) A horizontal asymptote b) A vertical asymptote at $x = 1$

c) A slant asymptote at $y = x$ d) No asymptote

e) None of these

25. Find $f(t)$ if $f'(t) = 2\cos(t) + \sec^2(t)$ and $f(\pi/3) = 4$.

a) $f(t) = 2\sin(t) + \frac{\sec^3(t)}{3} + \frac{4}{3} - \sqrt{3}$ b) $f(t) = 2\sin(t) + \tan(t) + 4 - 2\sqrt{3}$

c) $f(t) = -2\sin(t) + \frac{\sec^3(t)}{3} - \frac{4}{3}$ d) $f(t) = -2\sin(t) + \tan(t) + 4$

e) None of these

26. If $F(x) = \int_2^{x^2} \left(e^{t^2} - 1\right)\, dt$, then $F'(x) =$

a) $2x\left(e^{x^2} - 1\right)$ b) $2x\left(e^{x^4} - 1\right)$ c) $e^{x^4} - 1$

d) $e^{x^2} - 1$ e) None of these

27. Evaluate $\int x\cos(x^2)\,dx$.

a) Cannot be found
b) $2x\sin(x^2)$
c) $\frac{1}{2}\sin(x^2)+c$
d) $-\sin(x^2)$
e) None of these

28. The surface area of a spherical bloom is increasing at a rate of 22 ft^2/sec when its radius is 3 feet. Find the rate of change of its volume at that moment.

a) 32 ft^3/sec
b) 33 ft^3/sec
c) 34 ft^3/sec
d) 36 ft^3/sec
e) None of these

29. Let $f(x)=x^4-4x^3$. The graph of $y=f(x)$ has an inflection point at

a) $(2,-16)$
b) $(3,-27)$
c) $(4,0)$
d) No value of x
e) None of these

30. The most general solution for $\int x^3\sqrt{x}\,dx$ is

a) $\frac{2}{9}x^4\sqrt{x}$
b) $x^3\sqrt{x}$
c) $\frac{2}{9}x^4\sqrt{x}+c$
d) $x^4\sqrt{x}+c$
e) None of these

31. Evaluate $\int_1^2 (4x+5)(3x-2)\,dx$.

a) $\frac{47}{2}$
b) $\frac{51}{2}$
c) $\frac{61}{2}$
d) $\frac{57}{2}$
e) None of these

32. Evaluate $\int_1^2 \frac{2t+1}{t^2+t}\,dt$.

a) $\ln 3$
b) $\ln 2$
c) $\ln 6$
d) Cannot be found
e) None of these

33. Evaluate $\int_0^a x\sqrt{x^2+a^2}\,dx$, where $a > 0$.

a) $\frac{2\sqrt{2}-1}{3}a^2$ b) a^3 c) $\frac{2\sqrt{2}-1}{3}a^3$

d) $\frac{1}{3}a^2$ e) None of these

34. $\frac{d^2}{dx^2}(\ln(x)+2^x) =$

a) Cannot be found b) $2^x(\ln 2)^2 - \frac{1}{x^2}$ c) $2^x(\ln 2)^2$

d) $2^x \ln 2$ e) None of these

35. Evaluate $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$.

a) Cannot be found b) $-2\cos(\sqrt{x})+c$ c) $\sin(\sqrt{x})+c$

d) $2\cos(\sqrt{x})+c$ e) None of these

36. $\int_0^2 \frac{4x^3}{(x^2+1)^2}\,dx =$

a) Cannot be found b) $\frac{32}{25}$ c) $2\ln 5 - \frac{8}{5}$

d) $\ln 96$ e) None of these

37. The area between the curves $y = x^3$ and $y = \sqrt{x}$ is

a) $\frac{1}{2}$ b) $\frac{3}{4}$ c) $\frac{\sqrt{2}}{2}+1$

d) $\frac{5}{12}$ e) None of these

38. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, $y = 1$, and $x = 0$ about the x-axis.

a) 1 b) $\frac{4\pi}{5}$ c) $\frac{3\pi}{4}$

d) $\frac{\pi}{2}$ e) None of these

39. The volume of the solid obtained by rotating the region between the curves $y = x^2$, $y = 1$, and $x = 0$ about the line $y = 2$ is

a) $\frac{28\pi}{15}$ b) $\frac{\pi}{3}$ c) $\frac{5\pi}{3}$

d) $\frac{2\pi}{5}$ e) None of these

40. The temperature of a metal rod 8 meters long is $12/\sqrt{x+1}$ degrees Celsius at a distance x meters from one end of the rod. Find the average temperature of the rod.

a) 6° C b) 48° C c) 32° C

d) 4° C e) None of these